22137403

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

**MATHEMATICAL STUDIES**
**STANDARD LEVEL**
**PAPER 1**

Candidate session number

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | | | | | | | | |

Thursday 9 May 2013 (afternoon)

1 hour 30 minutes

Examination code

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 1 | 3 | – | 7 | 4 | 0 | 3 |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL*** *information booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

0120

Please **do not** write on this page.

Answers written on this page
will not be marked.

*Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.*

**1.** $z = \dfrac{17x^2}{a-b}$.

(a) Find the value of $z$ when $x = 12.5$, $a = 0.572$ and $b = 0.447$.
Write down your full calculator display. *[2 marks]*

(b) Write down your answer to part (a)

(i) correct to the nearest 1000;

(ii) correct to three significant figures. *[2 marks]*

(c) Write your answer to **part (b)(ii)** in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. *[2 marks]*

*Working:*

*Answers:*

(a) . . . . . .

(b) (i) . . . . . .

(ii) . . . . . .

(c) . . . . . .

**2.** $U = \{x \mid x \text{ is an integer, } 2 < x < 10\}$.

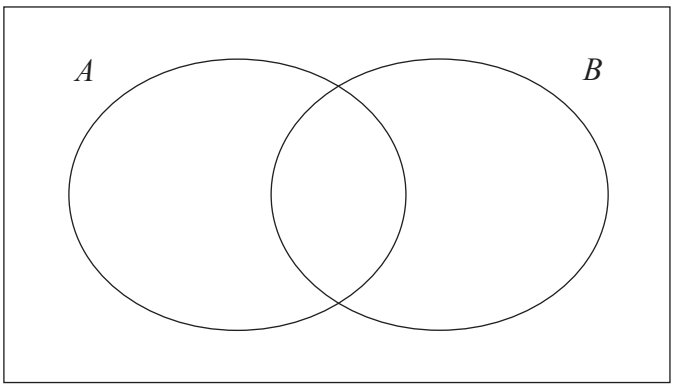
$A$ and $B$ are subsets of $U$ such that $A = \{\text{multiples of } 3\}$, $B = \{\text{factors of } 24\}$.

(a) List the elements of

(i) $U$;

(ii) $B$. *[2 marks]*

(b) Write down the elements of $U$ on the Venn diagram.




*[3 marks]*

(c) Write down $n(A \cap B)$. *[1 mark]*

*Working:*

*Answers:*

(a) (i) . . . . . .

(ii) . . . . . .

(c) . . . . . .

**3.** Consider the following logic propositions:

$p$ : *Yuiko is studying French.*
$q$ : *Yuiko is studying Chinese.*

(a) Write down the following compound propositions in symbolic form.

(i) Yuiko is studying French but not Chinese.

(ii) Yuiko is studying French or Chinese, but not both. *[3 marks]*

(b) Write down in words the **inverse** of the following compound proposition.

*If Yuiko is studying Chinese, then she is not studying French.* *[3 marks]*

*Working:*

*Answers:*

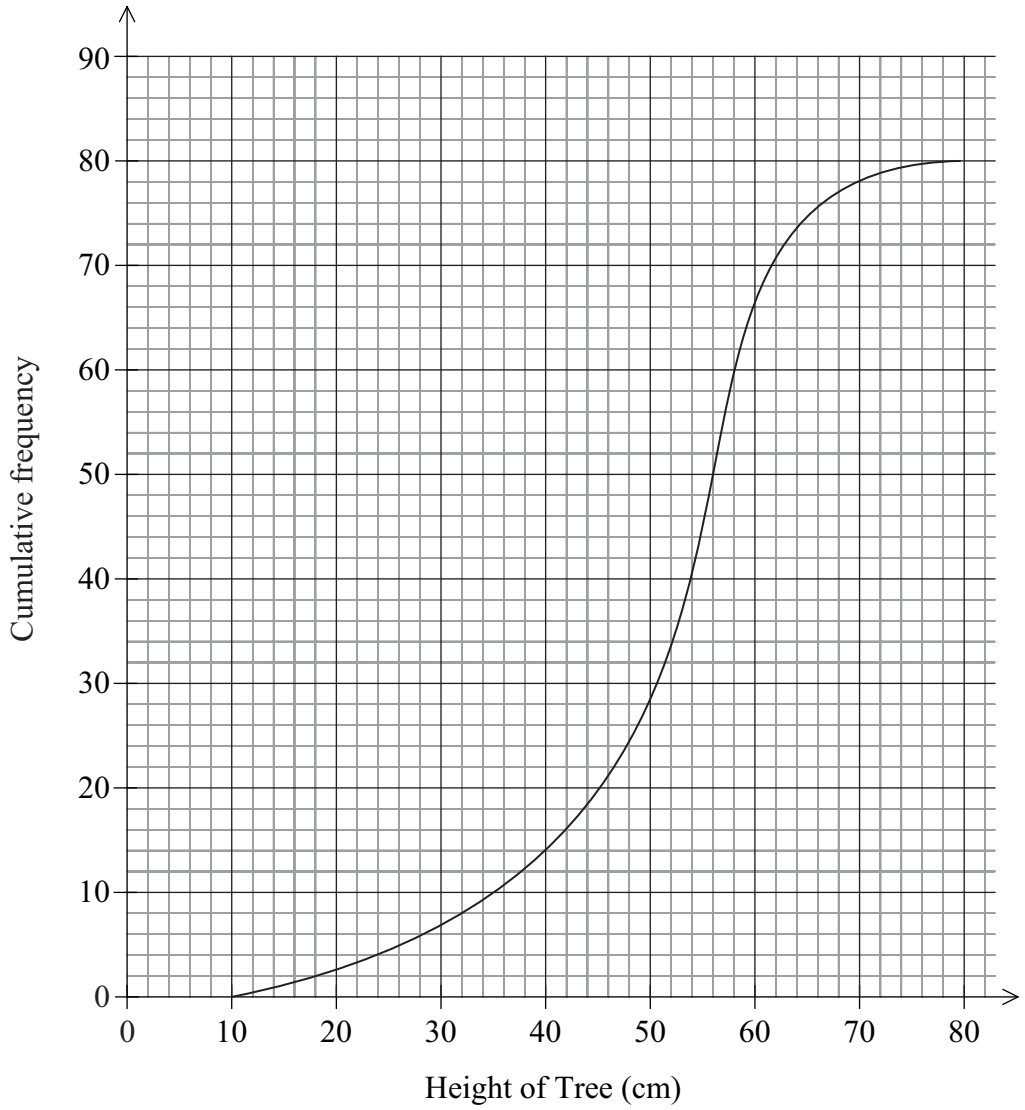
(a) (i) . . . . . .

(ii) . . . . . .

(b) . . . . . .

. . . . . .

**4.** The cumulative frequency graph shows the heights, in cm, of **80** young trees.




(a) Write down the median height of the trees. *[1 mark]*

(b) Write down the 75th percentile. *[1 mark]*

(c) Find the interquartile range. *[2 marks]*

(d) Estimate the number of trees that are more than 40 cm in height. *[2 marks]*

*(This question continues on the following page)*

*(Question 4 continued)*

*Working:*

*Answers:*

(a) . . . . . .

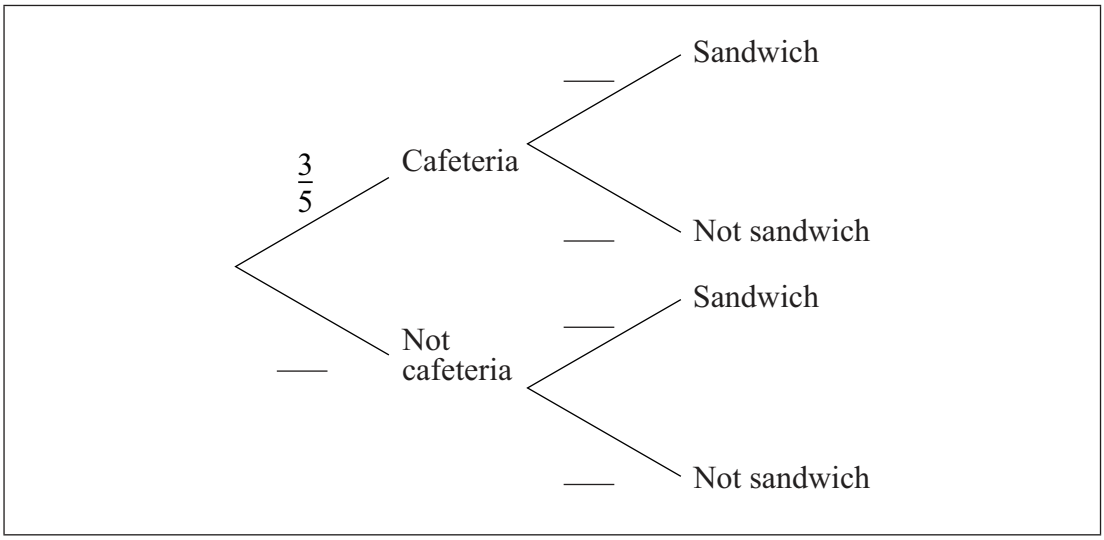
(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

**5.** The probability that Tanay eats lunch in the school cafeteria is $\frac{3}{5}$.
If he eats lunch in the school cafeteria, the probability that he has a sandwich is $\frac{3}{10}$.
If he does not eat lunch in the school cafeteria the probability that he has a sandwich is $\frac{9}{10}$.

(a) Complete the tree diagram below.




*[3 marks]*

(b) Find the probability that Tanay has a sandwich for his lunch. *[3 marks]*

*Working:*

*Answers:*

(b) . . . . . .

**6.** (a) Complete the truth table.

| $p$ | $q$ | $\neg p$ | $\neg p \vee q$ |
|---|---|---|---|
| T | T | | |
| T | F | | |
| F | T | | |
| F | F | | |

*[2 marks]*

Consider the propositions $p$ and $q$:

*p: x is a number less than 10.*
*q:* $x^2$ *is a number greater than 100.*

(b) Write in words the compound proposition $\neg p \vee q$. *[2 marks]*

(c) Using part (a), determine whether $\neg p \vee q$ is true or false, for the case where $x$ is a number less than 10 and $x^2$ is a number greater than 100. *[1 mark]*

(d) Write down a value of $x$ for which $\neg p \vee q$ is false. *[1 mark]*

*Working:*

*Answers:*

(b) . . . . . .

. . . . . .

(c) . . . . . .

(d) . . . . . .

**7.** The fourth term, $u_4$, of a geometric sequence is 135. The fifth term, $u_5$, is 101.25.

(a) Find the common ratio of the sequence. *[2 marks]*

(b) Find $u_1$, the first term of the sequence. *[2 marks]*

(c) Calculate the sum of the first 10 terms of the sequence. *[2 marks]*

*Working:*

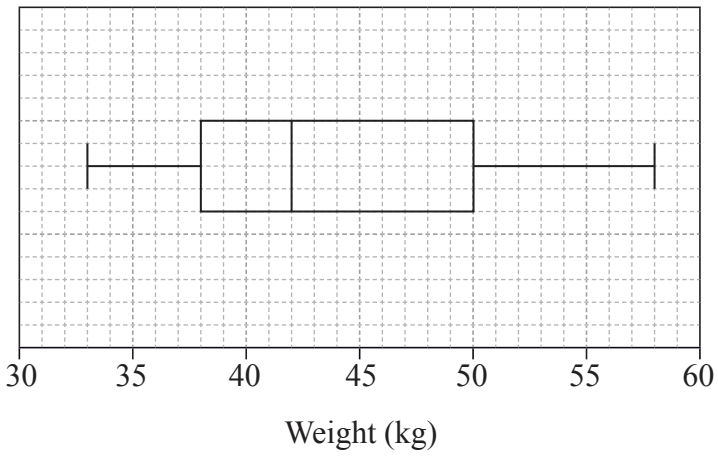
*Answers:*

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

**8.** The weights, in kg, of 60 adolescent females were collected and are summarized in the box and whisker diagram shown below.




(a) Write down the median weight of the females. *[1 mark]*

(b) Calculate the range. *[2 marks]*

(c) Estimate the probability that the weight of a randomly chosen female is more than 50 kg. *[1 mark]*

(d) Use the box and whisker diagram to determine if the mean weight of the females is less than the median weight. Give a reason for your answer. *[2 marks]*

*Working:*

*Answers:*

(a) . . . . . .

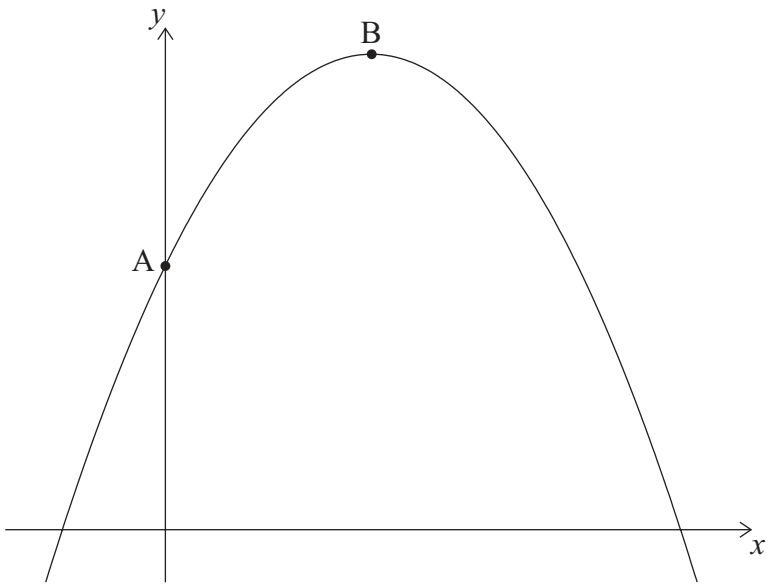
(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

. . . . . .

**9.** The graph of the quadratic function $f(x) = c + bx - x^2$ intersects the $y$-axis at point $\mathrm{A}(0, 5)$ and has its vertex at point $\mathrm{B}(2, 9)$.




(a) Write down the value of $c$. *[1 mark]*

(b) Find the value of $b$. *[2 marks]*

(c) Find the $x$-intercepts of the graph of $f$. *[2 marks]*

(d) Write down $f(x)$ in the form $f(x) = -(x - p)(x + q)$. *[1 mark]*

*Working:*

*Answers:*

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

**10.** The straight line, $L_1$, has equation $2y - 3x = 11$. The point A has coordinates $(6, 0)$.

(a) Give a reason why $L_1$ **does not** pass through A. *[1 mark]*

(b) Find the gradient of $L_1$. *[2 marks]*

$L_2$ is a line perpendicular to $L_1$. The equation of $L_2$ is $y = mx + c$.

(c) Write down the value of $m$. *[1 mark]*

$L_2$ **does** pass through A.

(d) Find the value of $c$. *[2 marks]*

*Working:*

*Answers:*

(a) . . . . . .

. . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

**11.** Neung is going home to Vietnam after working in Singapore.
She has 5000 Singapore dollars (SGD) and changes these into American dollars (USD) to take home.
The exchange rate between Singapore dollars (SGD) and American dollars (USD) is

$$1 \text{ USD} = 1.2945 \text{ SGD}.$$

There is also a 2.4 % commission on the exchange.

(a) Calculate the amount of commission on the exchange **in SGD**. *[2 marks]*

(b) Calculate the number of American dollars (USD) Neung takes home.
**Give your answer correct to 2 decimal places.** *[2 marks]*

At the airport in Vietnam, Neung changes 150 USD into Vietnamese dong (VND) to pay for her transport home.
The exchange rate between American dollars (USD) and Vietnamese dong (VND) is

$$1 \text{ USD} = 19\,495 \text{ VND}.$$

There is no commission.

(c) Calculate the number of Vietnamese dong that Neung receives. **Give your answer correct to the nearest thousand dong.** *[2 marks]*

*Working:*

*Answers:*

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

**12.** In an arithmetic sequence, the fifth term, $u_5$, is greater than the first term, $u_1$. The difference between these terms is 36.

(a) Find the common difference, $d$. *[2 marks]*

The tenth term of the sequence is double the seventh term.

(b) (i) Write down an equation in $u_1$ and $d$ to show this information.

(ii) Find $u_1$. *[4 marks]*

*Working:*

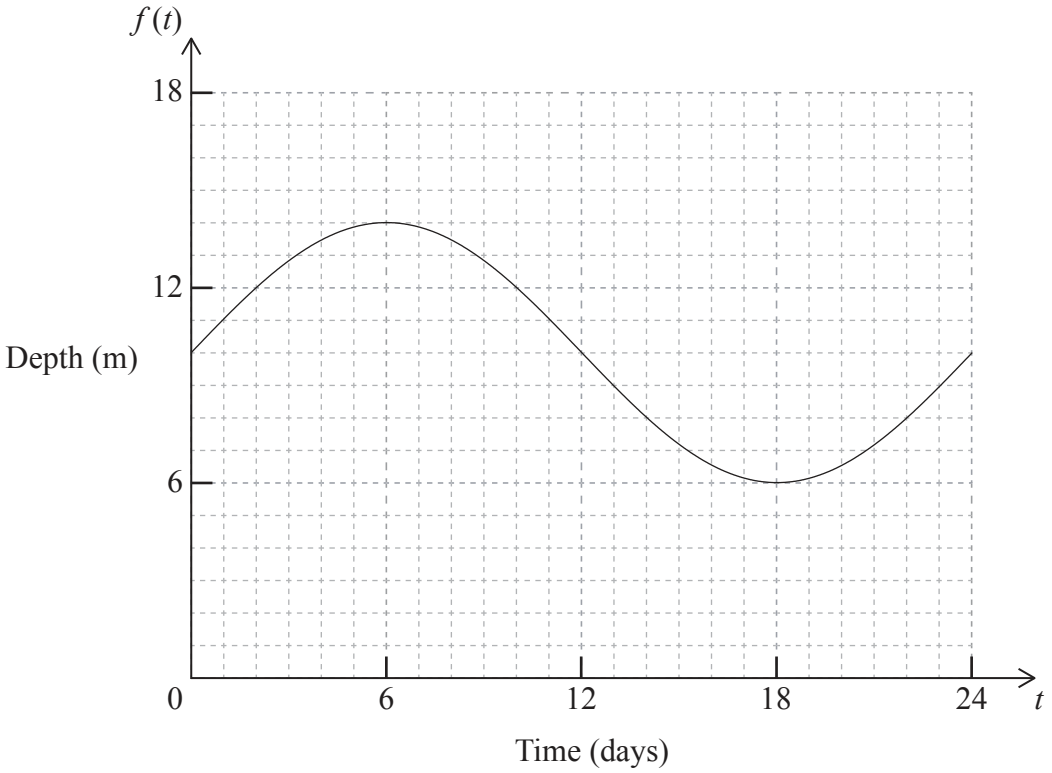
*Answers:*

(a) . . . . . .

(b) (i) . . . . . .

(ii) . . . . . .

**13.** The depth, in metres, of water in a reservoir during a 24 day period is shown on the graph.
The depth is modelled by $f(t) = c + k\sin(bt)$, where $t$ is the time in days and $bt$ is measured in degrees.




(a) Write down the value of $c$. *[1 mark]*

(b) Write down the value of $k$. *[1 mark]*

(c) Find the value of $b$. *[2 marks]*

(d) Write down the time interval for which the depth of water in the reservoir is decreasing. *[2 marks]*

*Working:*

*Answers:*

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

**14.** Jenny invested \$20 000 in a bank account that paid 3.5 % annual simple interest. She withdrew her investment from the account when its value was \$31 200.

(a) Find the number of years for which the money was invested. *[3 marks]*

Ramón invests \$18 000 in a bank account that pays 3.4 % nominal annual interest, **compounded quarterly**.

(b) Find the minimum number of years that Ramón must invest the money for his investment to be worth \$27 000. *[3 marks]*

*Working:*

*Answers:*

(a) . . . . . .

(b) . . . . . .

**15.** Consider the function $f(x) = ax^3 - 3x + 5$, where $a \neq 0$.

(a) Find $f'(x)$. *[2 marks]*

(b) Write down the value of $f'(0)$. *[1 mark]*

The function has a local maximum at $x = -2$.

(c) Calculate the value of $a$. *[3 marks]*

*Working:*

*Answers:*

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

2020